History of Mathematics: Euclid of Alexandria

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- Our Euclid is known as Euclid of Alexandria, since he taught and lived there primarily.
- The *Elements*, written by Euclid, is one of the oldest Greek mathematical treatise that’s been preserved in completeness, with the only exception being *On the Moving Sphere* by Autolycus of Pitane.
Although the *Elements* is what people most commonly associate with Euclid, there are other works by him, some of which have been lost. The ones that survived to present days:
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- Catoptrics: the authorship of this work is actually uncertain. It may have been the work of Theon of Alexandria.
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Purpose of the *Elements*

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- It is an introductory level textbook covering "higher arithmetic" or number theory, synthetic geometry and algebra.
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- Euclid made no claim of originality regarding the results in the book, but the organization and possibly some of the reasoning were his own.
The *Elements* is divided into thirteen books (chapters). First six books are on plane geometry, the next three on number theory, the tenth on incommensurables, and the last three on solid geometry.

A point is that which has no part.

A line is breadthless length.

A surface is that which has length and breadth only.
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A few examples:

- A point is that which has no part.
- A line is breadthless length.
- A surface is that which has length and breadth only.
Following the definitions, Euclid lists five postulates and five common notions. The five postulates are:

1. To draw a straight line from any point to any point.
2. To produce a finite straight line continuously in a straight line.
3. To describe a circle with any center and radius.
4. That all right angles are equal.
5. That, if a straight line falling on two straight lines makes the interior angles on the same side less than two right angles, the two straight lines, if produced indefinitely, meet on that side on which the angles are less than the two right angles.
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Common Notions

The five common notions:

1. Things which are equal to the same thing are also equal to one another.
2. If equals be added to equals, the wholes are equal.
3. If equals be subtracted from equals, the remainders are equal.
4. Things which coincide with one another are equal to one another.
5. The whole is greater than the part.
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Euclid made assumptions in his reasonings that would be considered as unjustified by today’s standard. Book I includes many well-known theorems we teach in high school geometry classes today, such as congruence triangle theorems, straight-edge and compass constructions, inequalities of angles and sides of triangles, parallel line properties etc.
Euclid proved the Pythagorean theorem without using the proportional properties of similar triangles. It was proven using the following figure:
The material in these two books of the *Elements* is believed to be largely from Hippocrates of Chios. They deal with the geometry of the circle. Again, they propositions are mostly similar to what one would find in our present day geometry textbook on circles.
Book V is on the theory of proportion, which is one of the most admired among the thirteen (the other one being Book X on incommensurables).
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- In Book VI, Euclid proved theorems concerning ratios and proportions related to similar triangles, parallelograms, and other polygons.
Although many people mistakenly think of the *Elements* as a purely geometric work, there are significant portions of it that deal with other topics. For example, Books II and V are algebraic in nature (although probably written with geometric application in mind). Books VII, VIII and IX are devoted to the theory of numbers.
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- The first two propositions in Book VII are essentially what we call the Euclidean algorithm in elementary number theory.
Prime and Perfect Numbers

Book VIII is said to be one of the less rewarding books of the *Elements*, dealing with some simple propositions on continued proportions and basic properties of square and cubes. Book IX, contains some very important results in number theory and their proofs.
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- Proposition 20: Prime numbers are more than any assigned multitude of prime numbers. Better known to us as the infinitude of primes.

- Proposition 35: If as many numbers as we please be in continued proportion, and there be subtracted from the second and the last numbers equal to the first, then as the excess of the second is to the first, so will the excess of the last be to all those before it.
Prime and Perfect Numbers

Last proposition: If as many numbers as we please, beginning from unity, be set out continuously in double proportion until the sum of all becomes prime, and if the sum is multiplied by the last, the product will be perfect. In other words, if

$$S_n = 1 + 2 + 2^2 + 2^3 + \cdots + 2^{n-1} = 2^n - 1$$

is a prime number, then $2^n - 1 (2^n - 1)$ is a perfect number.

Euclid did not say anything about the converse, but we know today that the converse is true if we only consider even perfect numbers. It is still an open question whether odd perfect number exists or not.

The first four perfect numbers (6, 28, 496 and 8128) are known to the Greeks.

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Book X of the *Elements* contains a systematic classification of incommensurable line segments of the form $a \pm \sqrt{b}$, $\sqrt{a} \pm \sqrt{b}$, $\sqrt{a} \pm \sqrt{b}$, and $\sqrt[4]{a} \pm \sqrt[4]{b}$, where $a$ and $b$ are commensurable.
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- Euclid himself considered this part of geometry rather than what we would consider today as algebra on irrational numbers.
- It contains 115 propositions, including things like rationalizing denominators.
- A geometric view using line segments was, at the time, more general due to the lack of a real-number system, since these line segments given by square roots or their combinations can easily be constructed by a straight-edge and compass.
Books XI, XII and XIII are on the topic of solid geometry.
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- Book XII, with 18 propositions, is mostly on the measurement of figures.

- The last book is devoted to properties of the five regular solids.